PRECIPITATION ASSOCIATED WITH A COLD FRONT, DECEMBER 7-9, 1956

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1. SYNOPTIC EVENTS

During the period December 7-9, 1956, a slow-moving cold front over the Central States brought prolonged precipitation to an area extending from northern Texas northeastward into New England (fig. 1). Amounts of over 3 inches accumulated in portions of southern Illinois and Indiana. Accumulations of 1 inch or more were reported from northeastern Oklahoma to Lake Erie.

Prior to this period, a cold front had moved through the Northern Plains States at about 15 miles per hour until 0630 GMT, December 6. It then became nearly stationary from Oklahoma to Illinois, although the portion north of Illinois continued an eastward movement. By 0630 GMT, December 7, the front extended along a line from southern Oklahoma through the eastern Great Lakes to a Low center in the St. Lawrence River Valley (fig. 2). A High pressure system moved southward behind the front and covered most of the Northern Plains States by 0630 GMT, December 7.

The upper air flow at 0300 GMT, December 7 (fig. 3), was marked by a broad belt of southwesterly winds at 500-mb. over all the United States except the Pacific Coast States. A trough lay from northern Idaho south-southwestward through central California and a large-amplitude ridge west of it extended northward into the Gulf of Alaska. The flow aloft east of the trough was remarkably free of curvature, and remained almost straight west-southwesterly over the Central States during the entire period of rainfall.

Several very minor waves formed along the front, but none developed significantly.

The surface High over the Northern Plains States had continued to intensify and, moving southward, had merged with an even stronger High pressure cell, also moving southward, which had formed in British Columbia. By 0630 GMT, December 9 (fig. 4) there was a 1055-mb. High near Salt Lake City, Utah, with a strong ridge extending southeastward.

During the 72 hours from 0630 GMT, December 6 to 0630 GMT, December 9, the portion of the front from Texas to the Great Lakes area moved only about 200 miles southeastward. One section, from western Tennessee to West Virginia, was almost stationary during the last 24 hours of that period. The southwestern section, however, maintained a slow southeastward drift until 1830 GMT, December 8, when it lay along a line from Nashville, Tenn. to Shreveport, La. and Del Rio, Tex. During the next 12 hours the portion of the front southwest of Nashville accelerated. By 0630 GMT on the 9th it lay across Mississippi and well east of Brownsville, Tex. This rapid frontal movement, accompanied by strong anticyclogenesis over the northern and western portions of the country, contributed significantly to the development of a surface Low in the East, which subsequently brought an end to the rainfall in the area of this study.

The 500-mb. chart had remained essentially unchanged from December 6 through December 8, with the long-wave trough in the West moving only as far as eastern Montana-Salt Lake City-Yuma by 0300 GMT on the 8th. However,

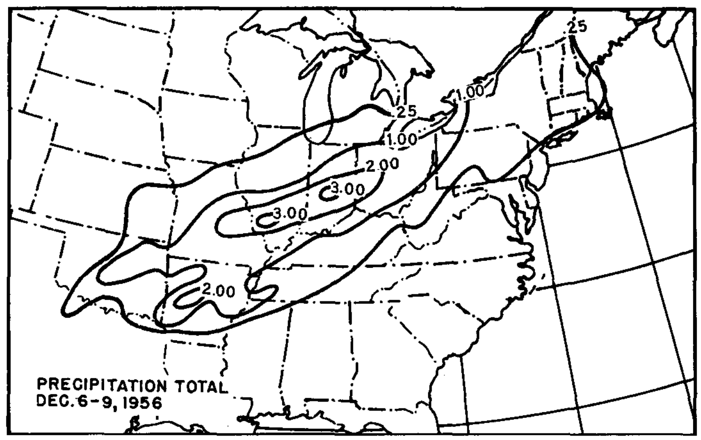

FIGURE 1.—Approximate total precipitation for the period 1230 GMT, December 6 through 1230 GMT, December 9, 1956. Units are inches.

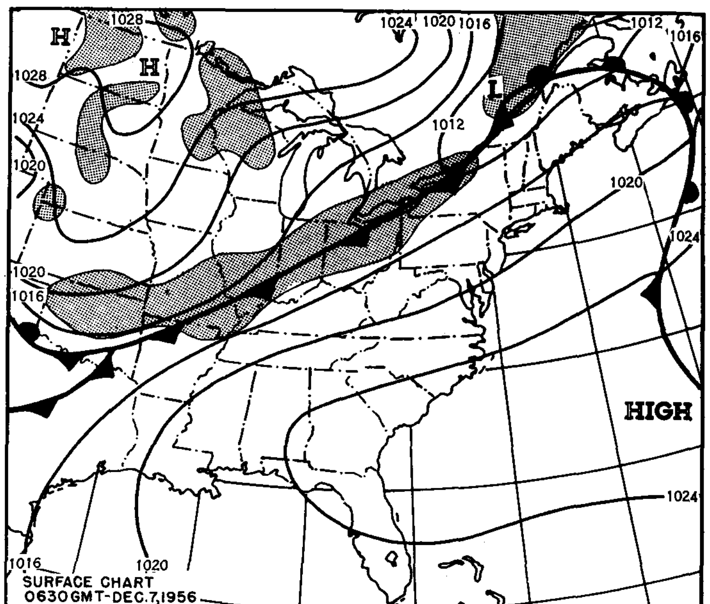

FIGURE 2.—Surface chart for 0630 GMT, December 7, 1956. Shaded areas indicate current precipitation.

by 0300 GMT, December 9 (fig. 5), the trough had accelerated eastward with a marked increase in wind speeds at 500 mb. over the Central States and the eastern Great Lakes-Ohio River Valley areas. This gave impetus to the surface Low forming in the East and by 0630 GMT, December 10, had effectively halted all precipitation in the eastern half of the country.

2. PHYSICAL PROCESSES RELATED TO LOW-LEVEL CONVERGENCE

In any prolonged precipitation situation such as the one being considered there is doubtless some low-level convergence occurring. Petterssen [1] has derived an equation which shows the factors related to the production

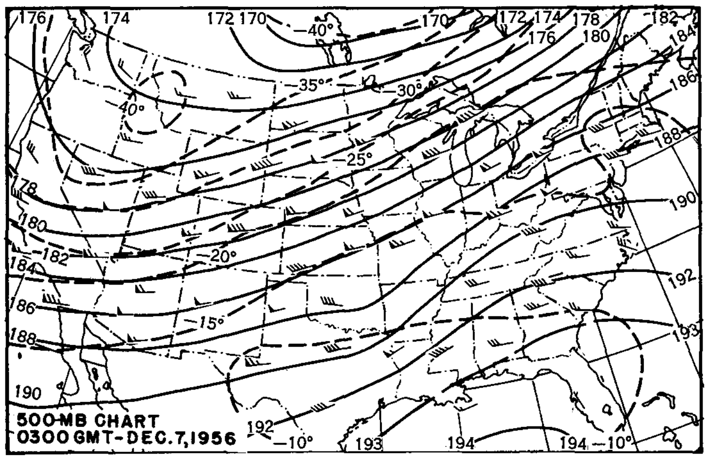

FIGURE 3.—500-mb. chart for 0300 GMT, December 7, 1956. Contours (solid lines) at 200-foot intervals are labeled in hundreds of feet. Isotherms (dashed lines) are at intervals of 5° C. Barbs on wind shafts are for speeds in knots (pennant=50 knots, full barb=10 knots, and half barb=5 knots).

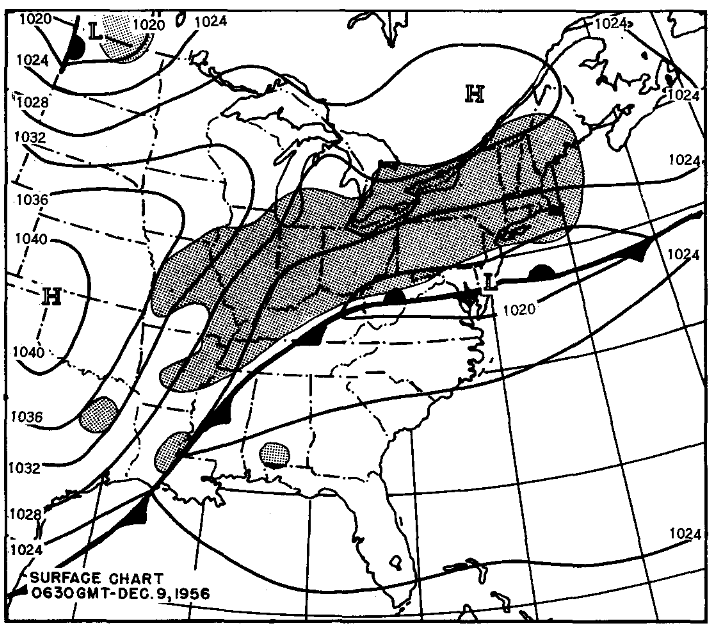

FIGURE 4.—Surface chart for 0630 GMT, December 9, 1956.

of low-level convergence. This investigation sought to study the terms of that equation as applied to this case. Means [2] has investigated a case of flood-producing rains in the Chicago area in this manner.

The development equation of Petterssen is:

$$-\eta_0 D_0 = A_\eta + \mathbf{V}_0\cdot\nabla\eta_0 - \frac{g}{f}\nabla^2 A_h - \frac{R}{f}\nabla^2\left[\ln\frac{p_0}{p}\overline{\omega(\Gamma_a-\Gamma)}\right] - \frac{R}{f}\nabla^2\left[\ln\frac{p_0}{p}\frac{1}{c_p}\overline{\frac{dW}{dt}}\right]$$

where subscript 0 designates a value at 1000 mb. and the bar an average value for the layer from 1000 mb. to the level of non-divergence. η is the vertical component of absolute vorticity; D is the divergence of the wind vector $\mathbf{V}$; A_η is the vorticity advection for the level of non-divergence; ∇ is the gradient vector operator in a constant-pressure surface; g is the acceleration of gravity; f is the Coriolis parameter; A_h is the thickness advection for the layer from 1000 mb. to the level of non-divergence; R is the gas constant; p is pressure; $\omega\equiv dp/dt$ is a measure of vertical motion (negative value indicates upward motion); $\Gamma_a\equiv dT/dp$ is the adiabatic lapse rate of temperature T with respect to p; $\Gamma\equiv\partial T/\partial p$ is the actual lapse rate with respect to p; c_p is specific heat at constant pressure; W is the heat (other than latent) supplied to a unit mass of air by non-adiabatic processes.

The terms in the development equation represent the following physical processes:

$-\eta_0 D_0$ is a measure of cyclonic development at 1000 mb.
A_η is vorticity advection at level of non-divergence.
$-\mathbf{V}_0\cdot\nabla\eta_0$ is vorticity advection at 1000 mb.

$-\frac{g}{f}\nabla^2 A_h$ is the development contribution of thickness advection for the layer 1000 mb. to level of non-divergence; it is proportional to the Laplacian of thermal advection for the layer.

$-\frac{R}{f}\nabla^2\left[\ln\frac{p_0}{p}\overline{\omega(\Gamma_a-\Gamma)}\right]$ is a "buoyancy term" and repre-

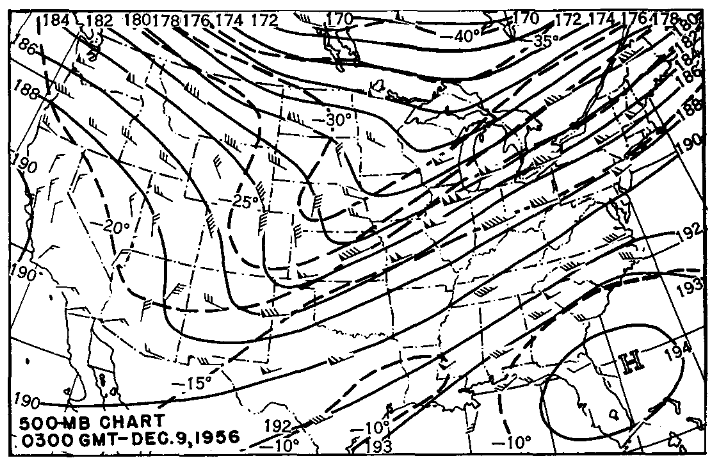

FIGURE 5.—500-mb. chart for 0300 GMT, December 9, 1956.

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sents the development contribution of local thickness changes due to adiabatic processes.

$-\frac{R}{f}\nabla^2\left[\ln\frac{p_0}{p}\frac{1}{c_p}\overline{\frac{dW}{dt}}\right]$ is the development contribution of local thickness changes due to non-adiabatic processes.

The terms on the right of the equation do not represent independent processes. Actually, the advective processes and the adiabatic and non-adiabatic influences are interacting to produce a very complex atmospheric process. Since the objective of this study is to determine whether or not the terms contributed to low-level convergence, and not to compare their magnitudes, they have been investigated separately. The level of non-divergence was not specifically located in this case, but was assumed to be near 500 mb. If it were as high as 300 mb., it is not likely that a change of sign would result in any of the terms of the equation.

In order to study the vorticity advection at the level of non-divergence (A_n), the vorticity at 500 mb. was computed using the finite difference method and a grid distance of 400 km. The vorticity field was then superimposed on the 500-mb. contours. It was apparent that only slight advection occurred on December 7 and 8, but it was positive in sign as the equation would require. On December 9, when the upper trough approached from the West (fig. 5), the vorticity advection increased but lagged considerably behind the sea level front. An examination of the 300-mb. charts added nothing in this case since the flow was essentially the same as that at 500 mb.

The second term of the equation, the vorticity advection at 1000 mb. ($-\mathbf{V}_o\cdot\nabla\eta_o$), is usually very small, according to computations by Petterssen. Examination of the sea level charts in this case (figs. 2 and 4) shows only slow development of cyclonic vorticity at low levels during most of the period of precipitation. The vorticity development came late in the period with the surface Low forming east of the area of heaviest rainfall. Consequently, this term was considered to be of little importance in the production of convergence.

The third term $\left(\frac{g}{f}\nabla^2 A_h\right)$, a measure of the effect of thermal advection, was studied by means of the 1000–500-mb. thickness charts. The thickness lines were superimposed on the sea level isobars, and the advection of the field was computed as the product of the geostrophic wind component and the thickness gradient measured over a distance of 100 nautical miles. Multiplication by a suitable factor gave the result as thermal advection in ° C. per 12 hours. The field of thermal advection was plotted on a chart and the Laplacian computed by finite differences over a 200-km. grid. Figure 6 contains a few representative charts giving the results of these computations.

The reliability of the computations of the Laplacian of thermal advection is naturally dependent upon the abundance of points at which it is possible to compute the advection. In the method used here points were chosen in all rectangles formed by the intersections of thickness lines and isobars. This resulted in 15 to 25 points being used at each upper air observation time. The areas of negative or positive Laplacian are fairly well defined by this density of points.

Figure 6A shows the area of negative values of the Laplacian of thermal advection extending from Little Rock, Ark., to Indianapolis, Ind., to Syracuse, N. Y. This area, where the thermal advection was contributing to low-level convergence, lies immediately ahead of the cold front. Isochrones of the onset of precipitation (not illustrated) show a steady progression from 1300 GMT, December 6 at Columbia, Mo., through 2100 GMT at Springfield, Ill., to 0100 GMT, December 7 at Indianapolis, Ind., and to 0500 GMT, at Dayton, Ohio.

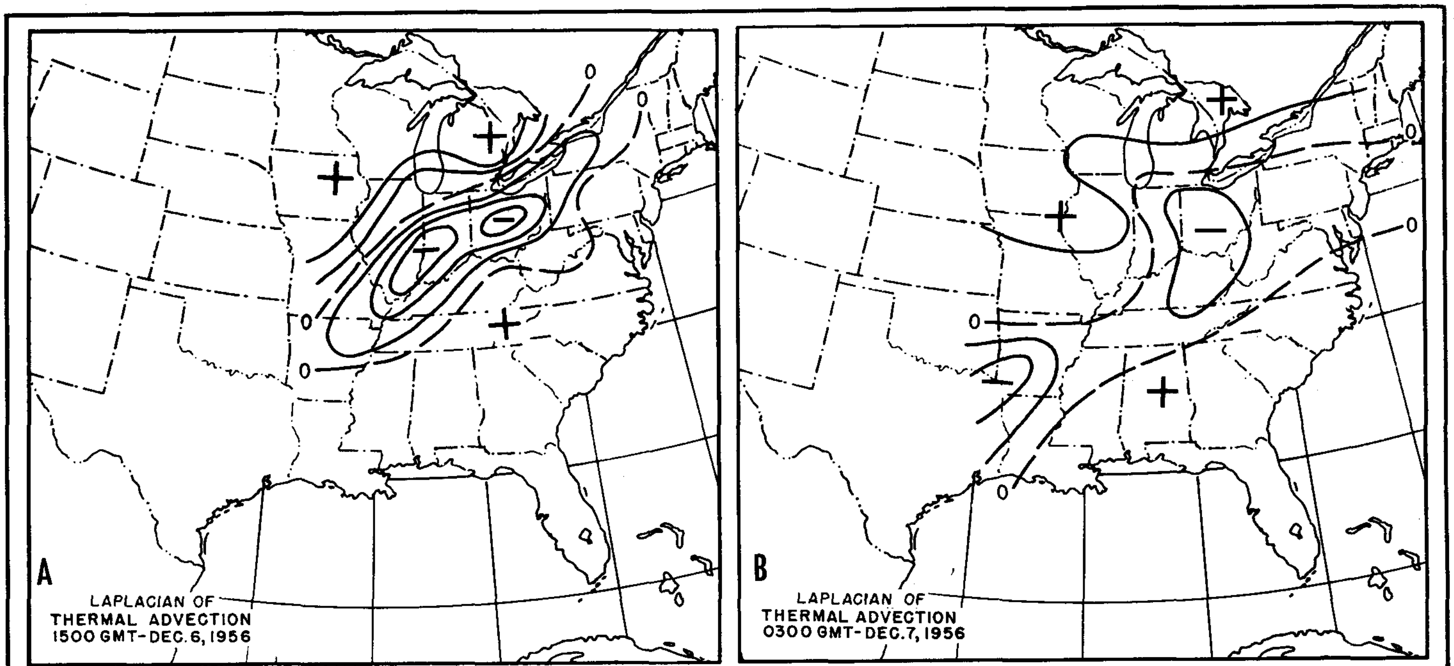
In figure 6B, the region of negative values of the Laplacian of thermal advection over Ohio corresponds very well with the area of precipitation. The correspondence is best between 0300 GMT, December 7 and mid-morning December 7 at such cities as Toledo, Dayton, Columbus, Cincinnati, and Akron.

In figure 6C, the region of negative values is again just ahead of the frontal position, and the time of precipitation at Little Rock, Ark., and Cairo, Ill., for example, corresponded rather well with the time of this chart.

The fourth term, called the "buoyancy term", is $-\frac{R}{f}\nabla^2\left[\ln\frac{p_0}{p}\overline{\omega(\Gamma_a-\Gamma)}\right]$. $\frac{R}{f}$ can be considered constant because there is only a small variation of latitude in the precipitation area. By choosing $p_o=1000$ mb. and $p=500$ mb., $\ln\frac{p_0}{p}$ also becomes a constant. The element to investigate is therefore reduced to the Laplacian of the average product $\overline{\omega(\Gamma_a-\Gamma)}$, which may be approximated by the product $\overline{\omega}\ \overline{(\Gamma_a-\Gamma)}$.

Vertical motion charts are routinely produced by the Joint Numerical Weather Prediction Unit (JNWP), and these were used for this parameter (fig. 7). It must be noted here that in this equation and in the accompanying figures, upward vertical motion is indicated by a negative value of $\overline{\omega}$. Also, vertical motion as computed by JNWP is the motion at the 500-mb. level rather than the average vertical motion from the surface to the level of non-divergence. It has been assumed in this case that vertical motion reaches a maximum value near the 500-mb. level, so that the average through the layer below would be less than that shown on the JNWP charts. However, the JNWP values are averaged over a large horizontal area, and literature on vertical motion [3] indicates values several times the magnitude of those on the computed charts. In this investigation, since only the direction of the motion, rather than its magnitude, was important there is little loss of accuracy in using the JNWP charts.

The stability index $\overline{(\Gamma_a-\Gamma)}$ while similar to a Showalter index [4], was computed by using the point on the sound-



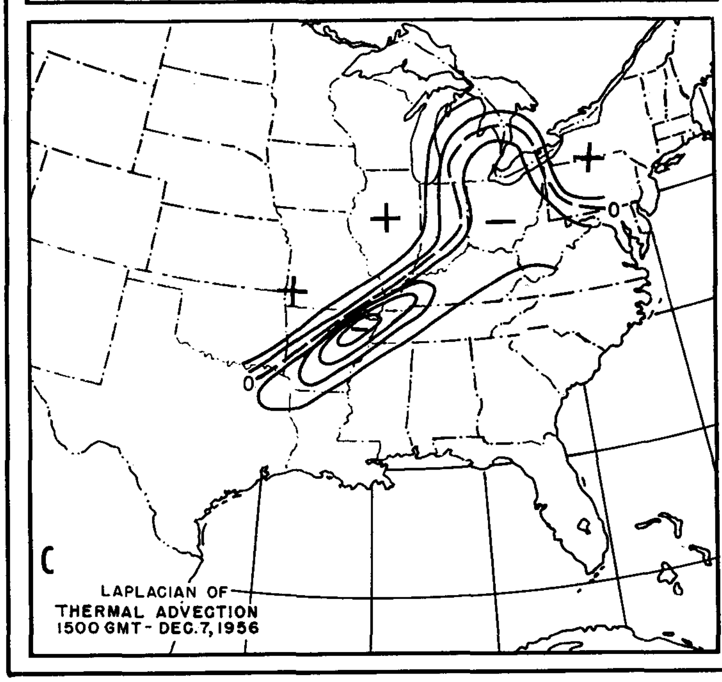


Figure 6.—Areas of minimum values of the Laplacian of thermal advection. (A) 1500 GMT, December 6; (B) 0300 GMT, December 7; and (C) 1500 GMT, December 7, 1956.

ing below 700 mb. which, when lifted to saturation, had a higher value of equivalent potential temperature than any other point so lifted. This index was then computed by lifting this point in the same manner as the 850-mb. point is lifted in a Showalter index.

Petterssen has shown that the sign of the entire buoyancy term is determined by whether or not the air is saturated. So long as the air remains unsaturated, any lifting will cool the air dry-adiabatically at a rate greater than the existing lapse rate. This results in the lifted air being subjected to a negative buoyancy force with respect to the ambient air, so that the term acts as a brake on the other processes. However, once the air does reach saturation, the sign of the term can be reversed with a consequent positive contribution to low-level convergence.

In investigating this term, it was therefore necessary to find areas of near-saturated conditions combined with areas of upward vertical motion and negative potential stability index. Potential stability indices were computed for the approximately 40 upper air stations in and around the area for the period from 0300 GMT, December 7, to 1500 GMT, December 8, and isopleths drawn (fig. 7). When the stability field was superimposed on the vertical motion field, it could be seen that the proper set of circumstances, i. e., upward vertical motion plus relative instability, existed over much of the area throughout the period. It therefore, remained only to determine where and when saturation occurred.

It was noted on inspection of the 850-mb. charts that, with the existing temperatures, a dew point of 10° C. over the area would result in near-saturated conditions. Study of the soundings showed that an increase of moisture occurred through a deep layer of air, at least as high as the 700-mb. level. In figure 8, the 10° C. isodrosotherm, the 850-mb. isotachs, and the axis of the maximum wind velocity at 850 mb. have been reproduced. The low-level jet carried the injection of moist air northeastward over the area. A close correlation was evident between the onset of rainfall and the time the 10° C. isodrosotherm reached a corresponding point. Several examples of this can be pointed out. Figures 8B and 8C show the 10° C. isodrosotherm progressing northeastward over southern

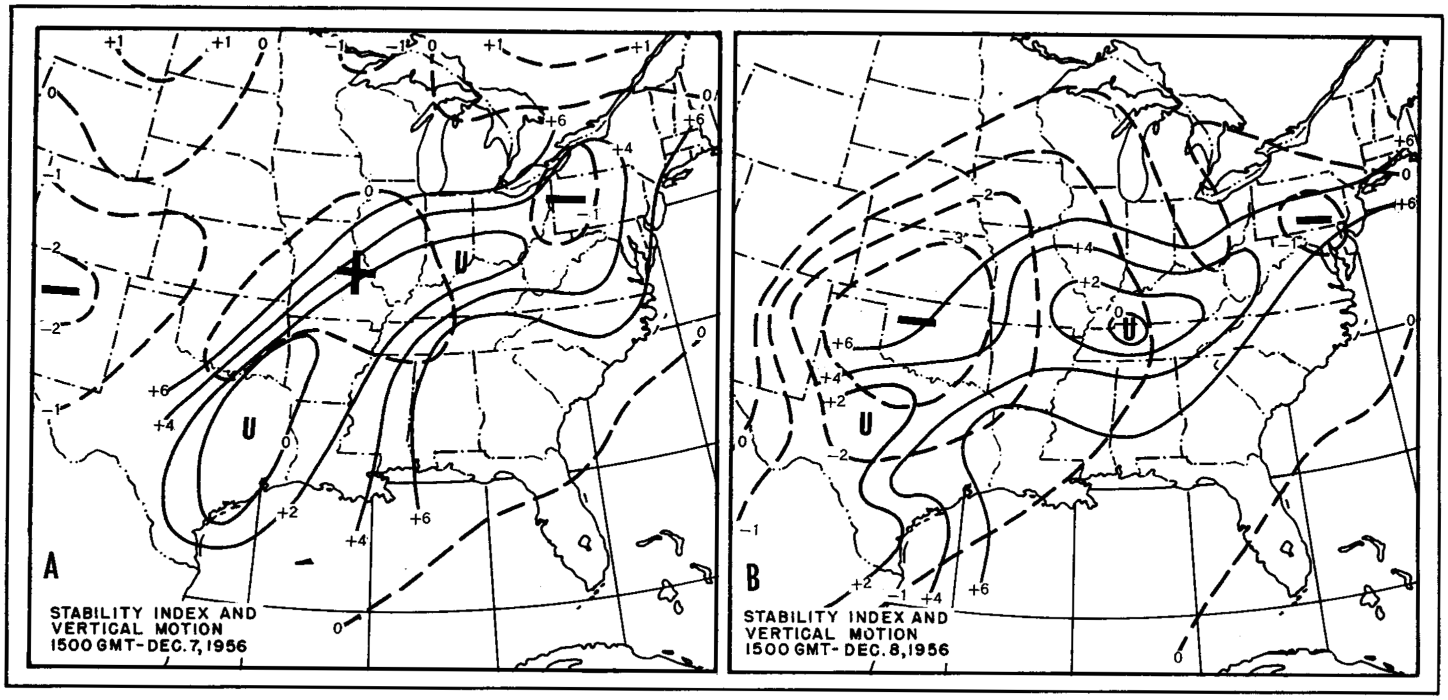


FIGURE 7.—Stability index (solid lines) with minus values indicating instability, and plus values, stability. Centers of instability are labeled U. Superimposed is the vertical motion pattern in cm/sec. (dashed lines) with minus values indicating upward motion. (A) 1500 GMT, December 7, and (B) 1500 GMT, December 8, 1956.

Missouri and southern Illinois into most of Indiana between the hours of 1500 GMT on December 6 and 0300 GMT on December 7. Rain began at Springfield, Mo. at 1700 GMT, December 6; at St. Louis, Mo. at 2100 GMT, December 6; at Ft. Wayne, and Indianapolis, Ind. at 0100 GMT, December 7. The rainfall began at each of these stations within an hour or two of the arrival of saturated conditions.

The final term of the equation is that of non-adiabatic contributions to low-level convergence, $-\frac{R}{f}\nabla^2\left[\ln\frac{p_0}{p}\frac{1}{c_p}\frac{\overline{dW}}{dt}\right]$. Petterssen has pointed out that this term is difficult to evaluate, but that statistical evidence shows it to be of importance. He also emphasizes that, by itself, the amount of heating or cooling is not important. Rather it is the configuration of the pattern of heating or cooling (i. e., the Laplacian) that determines the sign of the term and thus the sense of the contribution.

It can readily be seen that in cases of warming from below, as when a cold air mass moves over a warmer surface, there is a positive contribution to cyclonic development, while the reverse is true in an airmass being transported over a surface colder than itself.

In this case study, the air moving northeastward to the south of the front was passing over terrain of similar temperature and, consequently, was being neither cooled nor warmed from below. The southward-moving air, being relatively cold, was continually being transported over ground warmer than itself along the front. Consequently, there was a positive contribution to development along and immediately behind the front by the non-adiabatic influences.

To recapitulate, the terms of the development equation contributed to low-level convergence in the following ways:

The first term, vorticity advection at the level of non-divergence, is small, but positive in sign, thus contributing to low-level convergence.

The second term, vorticity advection at 1000 mb., was disregarded as being insignificantly small.

The third term, the Laplacian of thermal advection, was found to be generally negative in sign in areas just ahead of the cold front, thereby contributing positively to low-level convergence in those areas. There was good correlation between the negative areas of this term and areas of rainfall, both in time and space.

The fourth term, the buoyancy term, was evaluated qualitatively only, with positive contributions determined to be in regions of simultaneous occurrence of saturation, upward vertical motion, and negative potential stability index.

The correlation between time of near-saturated conditions (determined by the 10° C. isodrosotherm at 850 mb.) and the onset of rainfall was excellent. The correlation of areas of simultaneous occurrence of all three parameters mentioned above was fair to good in the early part of the period, and improved with time through the period.

The final term of the equation, relating to non-adiabatic influence, was determined to contribute positively in areas where cool air was moving over warmer terrain.

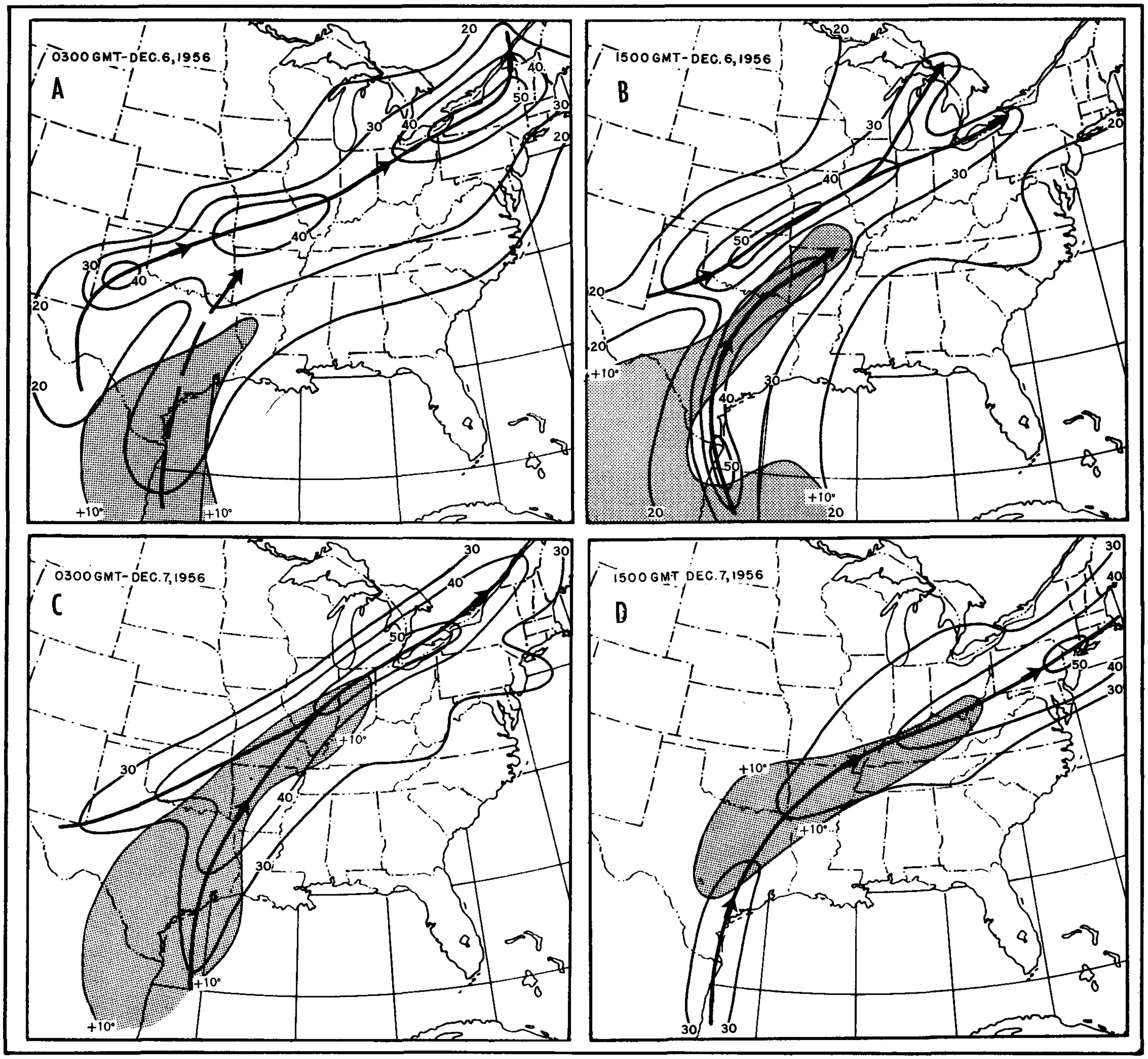


FIGURE 8.—850-mb. isotachs, maximum wind axis, and 10° C. isodrosotherm. Isotachs are labeled in knots at 10-knot invervals. Shading indicates areas where dewpoint at 850 mb. = 10° C. (A) 0300 GMT, December 6; (B) 1500 GMT, December 6; (C) 0300 GMT, December 7; and (D) 1500 GMT, December 7, 1956.

This was the case along and immediately behind the front. Ahead of the front this term could be neglected, as the air transport was over land surfaces with a nearly uniform temperature.

3. FORECASTING PROBLEM

The experienced forecaster recognizes several types of synoptic situations which produce prolonged precipitation in the Central States. One of these, a case of heavy thunderstorms in the warm air south of a front, has been studied by Means [2], and Nash and Chamberlain [5]. Oliver and Shaw [6] have studied a case of heavy warm-sector rains, while the situation described here involves precipitation closely related to a slow-moving cold front. To produce a satisfactory forecast in this case, the forecaster would have had to anticipate the slow speed of the frontal movement, the instability of the warm air ahead of the front, and the increase of moisture in the warm air.

Probably the key influence on the frontal movement was the stagnant nature of the large-scale waves in the upper air circulation. As shown in figure 3, the 500-mb. flow in the region of study was straight and parallel to the sea level front; therefore the contribution of vorticity advection aloft was insignificant. The movement of the front was then mainly due to low-level forces. It was not until an upper trough moved through the area (fig. 5) that the front changed its speed or shape to any significant degree.

In this case the instability and moisture increase in the warm air ahead of the front developed by the usual advective process. This is illustrated in figure 8 which shows the low-level wind increase and accompanying moisture advection characteristic to the air mass ahead of an advancing cold front in the Central States.

The question can be asked, given a situation such as this where the synoptic features are not unusual, how does the forecaster determine that precipitation will result in more than usual amounts? Unfortunately the answer to this question has been suggested here only in a qualitative way. The portion of this study which dealt with evaluation of the physical processes related to low-level convergence (precipitation) indicates that no complete quantitative evaluation of Petterssen's equation can come from that approach. It is hoped new research (c. f. [7]) will produce better methods for computing and interrelating precipitation with vertical motion, thermal advection, saturation effects, and the buoyancy features. Such methods are needed before those elements could be used in a routine forecast procedure. Even the present available methods are hampered in the area of this study by the lack of upper air observations. A reporting station at Evansville, Ind. would have added much to the reliability of this study. Until better methods and more abundant data are available, the forecaster, it appears, must rely on qualitative evaluation of the parameters related to precipitation formation.

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